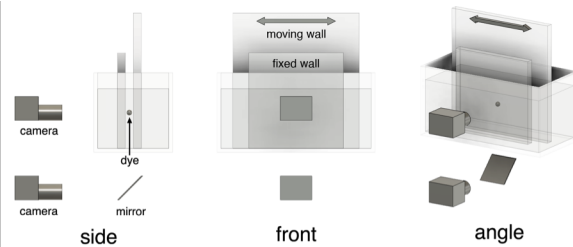


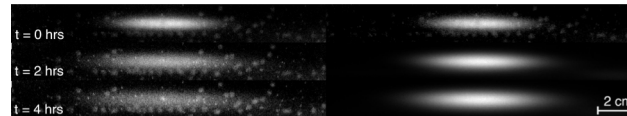
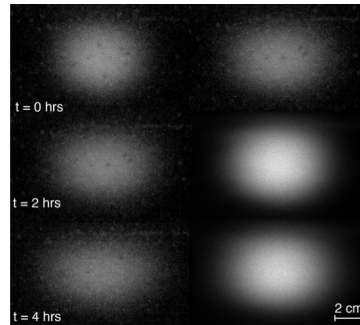
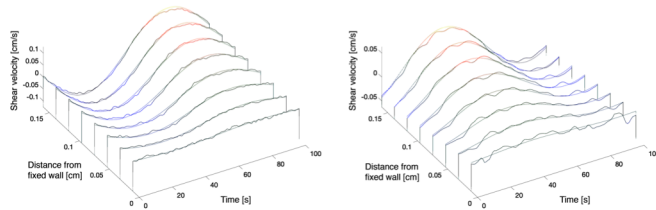
Mixing Induced by an Unsteady Shearing Wall Motion

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We present a study of shear dispersion. Here we consider the diffusing tracer advected by flows which are induced by a periodically oscillating wall in a linear stratified sodium chloride solution fluid between two infinite parallel walls. The figure below shows a schematic of the experimental setup from three different views.

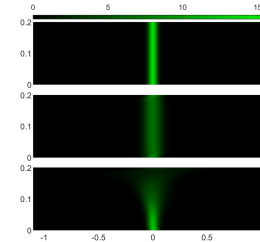
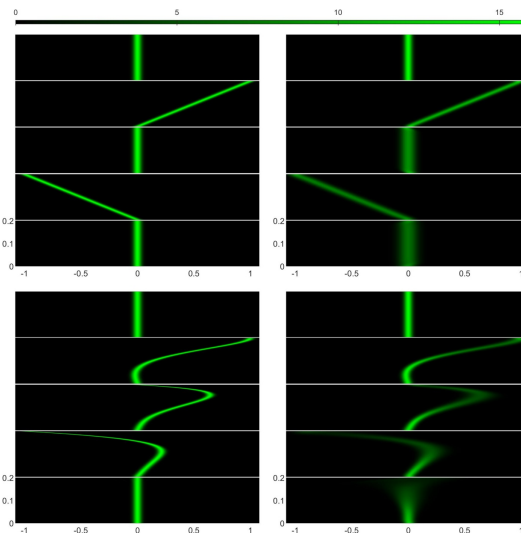


The flow induced by the wall motion was originally presented by Ferry[1,2]. The figure below shows the comparison of Particle Tracking Velocimetry (PTV) data with the Ferry wave analytical solution. Each slice corresponds with a time series of the shear velocity over a duration of one period taken at different distances between the fixed wall and the moving wall with gap thickness $L = 0.16$ cm. The velocity of the wall motion takes the form $v=A\omega \cos(\omega t)$. Left panel has wall oscillation amplitude $A = 2$ cm, right panel has $A = 1$ cm, other parameters: frequency $\omega = 2\pi * 0.01/s$, dynamic viscosity $\nu = 0.0113$ poise, and $L = 0.16$ cm.



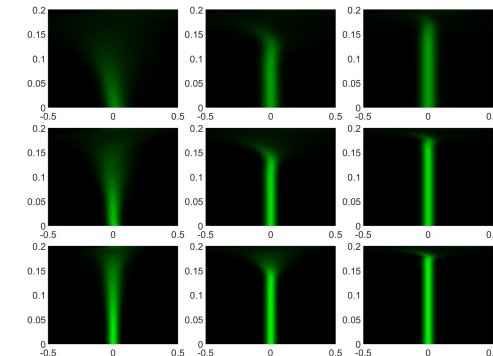
The figures above here show the experimental photos and Monte-Carlo simulations for the dye distribution viewed from the side at times $t = 0s$, $t = 7200s$, and $t = 14400s$ for the experimentally measured value of the diffusivity of Fluorescein in salt water of $\kappa = 3.3 \times 10^{-6} \text{ cm}^2/s$ and the same parameters in the Particle Tracking Velocimetry. The wall oscillation amplitude $A=1\text{cm}$ in the top figure, and $A=2\text{cm}$ in the bottom figure. If there were no flow, the cloud would have only spread by 0.3cm, while this corresponds a respective 19.3 fold increase, and 74.1 fold increase.

The figure below is a time series from a pseudo-spectral simulation for different viscosities and diffusivities, which shows the view of tracer from the top. Upper panels correspond to high viscosity $\nu = 10$ poise (linear shear like flow), while the lower panels correspond to the low viscosity $\nu = 0.01$ poise (nonlinear Ferry wave like flow). The left panels are computed with $\kappa = 0 \text{ cm}^2/s$, while the right panels utilize $\kappa = 10^{-5} \text{ cm}^2/s$.

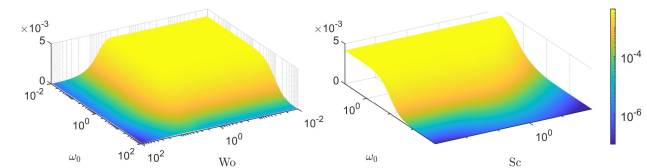


The left figure shows the top view of the tracer after one period of wall motion: the top panel has no flow, the middle has a linear shear, and the bottom panel has a nonlinear Ferry wave. The nonlinear Ferry wave causes the largest dispersion of the passive scalar. In addition to creating stronger overall dispersion, the nonlinear Ferry wave creates an enhanced vertical gradient which further enhances vertical transport.

The figure below shows the top view of the tracer after one period of wall motion for different diffusivities and viscosities. The viscosity decreases from left to right ($\nu = 0.01$ poise, 0.001 poise, 0.0001 poise) and the diffusivity decreases from the top to bottom ($\kappa = 5 \times 10^{-5} \text{ cm}^2/s$, $10^{-5} \text{ cm}^2/s$, $2 \times 10^{-6} \text{ cm}^2/s$). Note that the mixing is confined in a thin boundary layer for smaller viscosities.



Exact mathematical calculations provide an explicit formula for the enhanced diffusivity as a function of the Womersley number, $Wo = L\sqrt{\omega/\nu}$, the Schmidt number $Sc = \nu/\kappa$, and the dimensionless frequency. The left panel shows the enhanced dispersion as a function of Womersley number and dimensionless frequency $\omega_0 = \omega L^2 / \kappa$, while the right as a function of Schmidt and frequency at fixed unit Peclet number. Note that the Peclet scaling is purely quadratic for these flows. Also, observe that at fixed, high frequency, the right plot shows a mixing maximum as a function of Sc number.



We have shown that the sinusoidal motion of a wall can induce surprisingly strong and complex fluid mixing, particularly in the strongly nonlinear Ferry wave regime where a maximum effective diffusivity occurs as a function of viscosity which suggests an possible optimal mixing temperature.

Acknowledgements

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